

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (27)

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Question No (27)

(i) Prove the identity $\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} \equiv \frac{\tan \theta - 1}{\tan \theta + 1}$.

(ii) Hence solve the equation $\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{\tan \theta}{6}$, for $0^\circ \leq \theta \leq 180^\circ$.

Solution

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$$\begin{aligned}
 \textcircled{i} \quad \text{R.H.S} &= \frac{\tan \alpha - 1}{\tan \alpha + 1} \\
 &= \frac{\frac{\sin \alpha}{\cos \alpha} - 1}{\frac{\sin \alpha}{\cos \alpha} + 1} \\
 &= \frac{\sin \alpha - \cos \alpha / \cos \alpha}{\sin \alpha + \cos \alpha / \cos \alpha} \\
 &= \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} \\
 &= \text{L.H.S}
 \end{aligned}$$

$$\textcircled{ii} \quad \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} = \frac{\tan \alpha}{6} \quad 0 \leq \alpha \leq 180^\circ$$

$$\frac{\tan \alpha - 1}{\tan \alpha + 1} = \frac{\tan \alpha}{6} \quad \text{---} \quad \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} = \frac{\tan \alpha - 1}{\tan \alpha + 1}$$

$$6(\tan \alpha - 1) = \tan \alpha (\tan \alpha + 1) \quad \text{From (i)}$$

$$6 \tan \alpha - 6 = \tan^2 \alpha + \tan \alpha$$

$$\tan^2 \alpha + \tan \alpha - 6 \tan \alpha + 6 = 0$$

$$\tan^2 \theta - 5 \tan \theta + 6 = 0$$

By factorization

$$\tan^2 \theta - 3 \tan \theta - 2 \tan \theta + 6 = 0$$

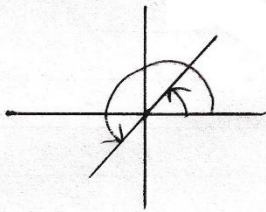
$$\tan \theta (\tan \theta - 3) - 2 (\tan \theta - 3) = 0$$

$$(\tan \theta - 3)(\tan \theta - 2) = 0$$

$$\tan \theta - 3 = 0$$

$$\tan \theta = 3$$

$\tan$ is +ve in I
and III quadrant



Basic angle

$$\alpha = \tan^{-1}(3)$$

$$\alpha = 71.6^\circ$$

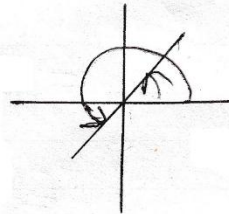
$$\theta = \alpha,$$

$$\theta = 71.6^\circ$$

$$\tan \theta - 2 = 0$$

$$\tan \theta = 2$$

$\tan$ is +ve in I
and III quadrant



Basic angle

$$\alpha = \tan^{-1}(2)$$

$$\alpha = 63.4^\circ$$

$$\theta = \alpha$$

$$\theta = 63.4^\circ$$

For the given range

$$\theta = 63.4^\circ, 71.6^\circ$$

