

Cambridge International AS & A Level

Mathematics

9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (28)

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Question No (28)

A tourist attraction in a city centre is a big vertical wheel on which passengers can ride. The wheel turns in such a way that the height, h m, of a passenger above the ground is given by the formula $h = 60(1 - \cos kt)$. In this formula, k is a constant, t is the time in minutes that has elapsed since the passenger started the ride at ground level and kt is measured in radians.

(i) Find the greatest height of the passenger above the ground.

One complete revolution of the wheel takes 30 minutes.

(ii) Show that $k = \frac{1}{15}\pi$.

(iii) Find the time for which the passenger is above a height of 90 m.

Solution

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(i)

$$h = 60(1 - \cos kt)$$

height will be greatest when $\cos kt$ has least value i.e. -1

$$\begin{aligned} \Rightarrow \text{greatest height, } h &= 60(1 - (-1)) \\ &= 60(1 + 1) \\ &= 120 \text{ m} \quad \underline{\underline{\text{Ans}}} \end{aligned}$$

(ii)

As given

when $t = 30$, angle $kt = 2\pi$ radians.

$$\Rightarrow k(30) = 2\pi$$

$$k = \frac{2\pi}{30}$$

$$k = \frac{\pi}{15}$$

proved.

(iii)

$$\text{As } h = 60\left(1 - \cos \frac{\pi t}{15}\right) \quad \text{--- } k = \frac{\pi}{15} \quad \text{(ii)}$$

when $h = 90$

$$\Rightarrow 90 = 60\left(1 - \cos \frac{\pi t}{15}\right)$$

$$1 - \cos \frac{\pi t}{15} = \frac{90}{60}$$

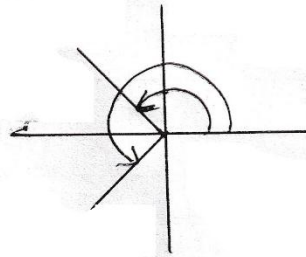
$$\cos \frac{\pi t}{15} = 1 - \frac{90}{60}$$

$$\cos \frac{\pi t}{15} = 1 - \frac{3}{2}$$

$$= \frac{2-3}{2}$$

$$\cos \frac{\pi t}{15} = -\frac{1}{2}$$

$\cos$ is -ve in III and II quadrant



basic angle

$$\alpha = \cos^{-1}(\frac{1}{2})$$

$$\alpha = \pi/3$$

$$\Rightarrow \frac{\pi t}{15} = \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

$$t = \frac{2\pi}{3} \times \frac{15}{\pi}$$

$$t = 10 \text{ minutes}$$

$$\frac{\pi t}{15} = \pi + \frac{\pi}{3}$$

$$\frac{\pi t}{15} = \frac{4\pi}{3}$$

$$t = \frac{4\pi}{3} \times \frac{15}{\pi}$$

$$t = 20 \text{ minutes}$$

$\therefore$ duration of time for which the passenger is above 90 m = $20 - 10$
 $= 10 \text{ min}$ Ans

