Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (29)

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Question No (29)

(i) Prove the identity $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 \equiv \frac{1-\cos x}{1+\cos x}$.

(ii) Hence solve the equation $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \frac{2}{5}$ for $0 \leqslant x \leqslant 2\pi$.

Solution

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(i)

$$L.H.S = \left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2$$

$$= \left(\frac{1}{\sin x} - \frac{1}{\frac{\sin x}{\cos x}}\right)^2$$

$$= \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)^2$$

$$= \left(\frac{1-\cos x}{\sin x}\right)^2$$

$$= \frac{(1-\cos x)^2}{\sin^2 x}$$

$$= \frac{(1-\cos x)^2}{1-\cos^2 x} \qquad \sin^2 x = 1-\cos^2 x$$

$$= \frac{(1-\cos x)^2}{(1-\cos x)(1+\cos x)} \qquad \because a^2-b^2 = (a-b)(a+b)$$

$$= \frac{1-\cos x}{1+\cos x}$$

$$= R.H.S$$

(ii) $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \frac{2}{5}$ for $0 \le x \le 2\pi$

$\Rightarrow \frac{1-\cos x}{1+\cos x} = \frac{2}{5}$ from part (i)

$5(1-\cos x) = 2(1+\cos x)$

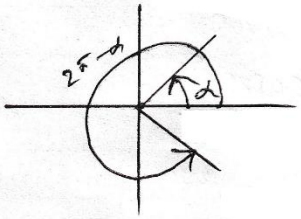
$5 - 5\cos x = 2 + 2\cos x$

$5 - 2 = 2\cos x + 5\cos x$

$3 = 7\cos x$

$\cos x = 3/7$

cosine is +ve in I and IV quadrant



Basic angle

$\alpha = \cos^{-1}(3/7)$

$= 1.13$ radians

$\therefore x = \alpha, \ 2\pi - \alpha$

$= 1.13, \ 2\pi - 1.13$

$x = 1.13, \ 5.15$ radians