Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 5-Trigonometry

Question No (30)

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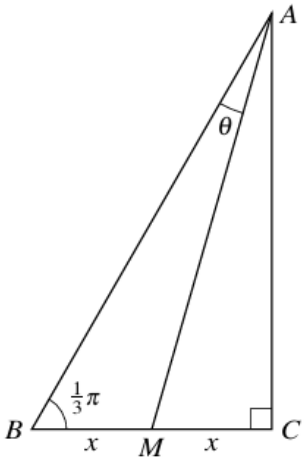


In the diagram, triangle ABC is right-angled at C and M is the mid-point of BC. It is given that angle $ABC = \frac{1}{3}\pi$ radians and angle $BAM = \theta$ radians. Denoting the lengths of BM and MC by x,

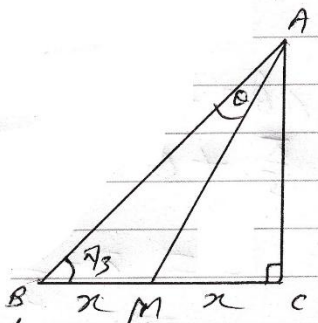
(i) find AM in terms of x,

(ii) show that $\theta = \frac{1}{6}\pi - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$.

Solution

On Next page

(i) From the $\triangle ABC$



$$\tan\sqrt{3} = \frac{AC}{BC}$$

$$\sqrt{3} = \frac{AC}{2x}$$

$$AC = 2\sqrt{3}\,x$$

On $\triangle AMC$, using pythagoras

$$(AM)^2 = (AC)^2 + (MC)^2$$

$$= (2\sqrt{3}x)^2 + (x)^2$$

$$= 4(3)x^2 + x^2$$

$$= 12x^2 + x^2$$

$$(AM)^2 = 13x^2$$

$$\therefore AM = \sqrt{13}\,x$$

(ii) On $\triangle AMC$

$$\tan M\hat{A}C = \frac{\text{vertical}}{\text{Base}}$$

$$= \frac{MC}{AC}$$

$$= \frac{x}{2\sqrt{3}x}$$

$$\tan M\hat{A}C = \frac{1}{2\sqrt{3}}$$

$$m\hat{A}C = \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

In $\triangle ABC$

$$A\hat{B}C + B\hat{A}M + m\hat{A}C + m\hat{C}A = \pi$$

$$\frac{\pi}{3} + \theta + \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right) + \frac{\pi}{2} = \pi$$

∴ Sum of angles in $\triangle$ is π

$$\theta = \pi - \frac{\pi}{3} - \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

$$= \frac{\pi}{2} - \frac{\pi}{3} - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

$$= \frac{3\pi - 2\pi}{6} - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

$$\theta = \frac{\pi}{6} - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right) \quad \text{proved}$$